Optimized abrasive waterjet nozzle design using genetic algorithms

J. Schwartzentruber
Ryerson University Department of Mechanical and Industrial Engineering

C. Narayanan
ASCOMP AG

H.-T. Liu
OMAX Corp.

M. Papini
Ryerson University Department of Mechanical and Industrial Engineering

Abstract

Abrasive waterjet (AWJ) performance and application is heavily dependent on nozzle geometry and operating conditions. The current research presents a novel technique, using genetic algorithms (GAs), to optimize a nozzle design and operating conditions for abrasive waterjet machining (AWJM) operations. The optimization program uses a one-dimensional three-phase flow model to predict exiting particle velocity (+/- 5% error). The model takes into account the influence of the mixing tube diameter and length, orifice diameter, abrasive feed tube diameter and length, mixing chamber diameter and length, hydraulic pressure, abrasive flow rate, density and viscosity of both air and water, density of abrasive, discharge coefficient of the nozzle, ambient pressure and temperature, and abrasive size distribution. The optimization program modifies the input parameters of the particle velocity model, using a GA scheme, in order to optimize the abrasive kinetic energy for a given set of operating conditions. The optimization algorithm is constrained by some rules of thumb in order to ensure realistic nozzle behavior and performance. The results of the optimization program are compared with a previous nozzle design based on their theoretical cutting performance over a range of abrasive loadings. The results clearly demonstrate the potential for efficiency gains. The gains provided by the optimized nozzle designs vary depending on the operating conditions, where the optimized designs cut 16% faster for the same hydraulic power requirements. The results show a trend that the gain in efficiency shows a declining tendency with increase in abrasive loading.

Nomenclature

A	Orifice cross-sectional area
c	Constant
c_d	Discharge coefficient
E_a	Average abrasive energy
f_O	Fitness value
$\dot{m}_a$	Mass abrasive flow rate
N_d	Mixing tube diameter
N_L	Mixing tube length
O_d	Orifice diameter
P	Hydraulic power
p	Pressure
p_d	Mean particle diameter
P_{BL}	Baseline hydraulic power
Q	Volumetric Flow rate
V_{Pavg}	Average particle velocity
ρ	Density of water

1. INTRODUCTION

The abrasive waterjet (AWJ) nozzle is a critical component in the efficiency and efficacy of the abrasive waterjet machining (AWJM) operation. A typical AWJ nozzle consists of an orifice, mixing chamber, abrasive feed port and mixing tube. The orifice generates a high velocity single phase water jet into the mixing chamber, which is then injected with air and abrasive via the abrasive feed port, which is then accelerated down the length of the mixing tube. The resulting exiting stream is typically a supersonic 3 phase jet with high erosive capabilities. Several authors have investigated the influence of nozzle components and their effect on various machining responses. Chen and Geskin [1] have shown that increasing orifice diameter increases the depth of cut, with the derivative of the depth-of-cut/orifice diameter curve indicating that depth-of-cut rate decreases with increase in orifice diameter. This decrease in depth of cut efficiency with increase in diameter has also been observed by Blickwedel [2] and Guo et al. [3]. Guo et al. [3] showed that there is an optimal orifice diameter, which is dependent on balance between orifice size and pump pressure.

Conversely, researchers have shown that increasing the diameter of the mixing tube results in a decrease in the depth of cut in brittle materials [4], however, there exists an optimal diameter for ductile behaving substrates [2]. This decrease in depth of cut has been attributed to reduced density of the abrasive-air-mixture [2] and increased turbulence [5]. However, small diameter mixing tubes promote increase particle collision, friction and abrasive fragmentation [6], which hinders the acceleration process. Kovacevic [6] and Blickwedel [2] have shown that there is an optimal focus diameter that is dependent on pressure [2], abrasive particle diameter [7], abrasive flow rate [2] and target properties [5].

Blickwedel [2] and Laurinat et al. [8] have shown that relationship between mixing tube length and depth of cut is linearly increasing length until an optimal value. Further increases in the mixing tube length results in a decrease in depth of cut due to increases in friction. It has been shown that the optimal focus length is dependent on pump pressure [9], stand-off distance [8] and mass flow rate [5].

Beyond the geometric consideration of the nozzle design, operating conditions also play an import role in the erosive ability of the AWJ. Raju and Srinivasa [10] have shown that their exists and optimal abrasive flow rate, where very high abrasive flow rates generate excessive drag, negating the benefits of increased particle impact. The optimal abrasive flow rate is dependent on pump pressure [3, 4, 9], orifice diameter [4, 5], mixing tube length [5], particle diameter [5, 11] and abrasive type [5, 11]. Similarly, abrasive particle diameter has an effect on depth of cut and has an optimal configuration [12-14]. Pump pressure has a general trend of increasing pump pressure results in an increase in depth of cut [5, 15].

Based on the competing factors between/within geometric properties of the nozzle and operating conditions, determination of the optimal machining configuration is highly complex. AWJ optimization is well studied but is primarily limited to empirical studies (mostly Taguchi or factorial design of experiments), for select materials and relatively few machining configuration factors [16-24]. Several authors have numerically solved for the flow field inside the nozzle and out, however, these studies focused on various flow characteristics rather than optimization [25-29]. This is largely due to the complex flow structure of the AWJ, of which conventional computation fluid dynamics is unreasonable for optimization problems due to the excessive computation time.

The proposed model uses genetic algorithm (GA) optimization and takes advantage of a high accuracy and rapid solving of a one-dimensional numerical model for calculating abrasive particle energy based on nozzle geometry and operating conditions. The current model is beneficial since it considers all machine configuration factors and their interacting effects using a purely analytical approach.

2. MODEL DEVLEOPMENT

The foundation of the optimization model is the waterjet energy code developed by Narayanan et al. [30]. The waterjet energy code is a one-dimensional computational fluid dynamic model, designed specifically for AWJ injection systems. The model utilizes 23 input factors to describe the geometry of the nozzle and operating conditions (discussed in Section 2.2). The GA program then varies the nozzle geometry and operating conditions and injects them into the waterjet energy code. The waterjet energy model solves the system for the given input configuration and outputs various exit flow characteristics (i.e. exit abrasive velocity, water velocity, volume fractions, abrasive energies, etc.). These flow metrics are then given back to the optimization algorithm, which then determines the fitness of the configuration with respect to the objective function and model constraints (Section 2.3). The program continues to iterate until convergence. The typical run time is 2.8 hours, in which the program considers 1829 nozzle/operating conditions configurations using an i7 4790K CPU, operating at 4 GHz.

2.1. Program structure

Figure 1 outlines the optimization program structure. The optimization program was built with MATLAB R2014b (MathWorks, MA, USA) using the built in GA optimization function and settings. The genetic algorithm is supplied with a set of upper and lower bounds within which it can vary the input factors. Remaining within the set of upper and lower bounds, the GA randomly develops the initial population of nozzle configurations (Block A in Figure 1). The details pertaining to the GA program will be discussed in Sections 2.2 and 2.3. The number of nozzle configurations is controlled by the population setting (set to 30). The program then injects the input file into the waterjet energy code, which solves various flow metrics unique to the prescribed inputs (Block B in Figure 1). The program then extracts the average abrasive energy value from the waterjet energy code and reads it into the objective function of the genetic algorithm (Block C in Figure 1). The program then evaluates the results to ensure that it meets the prescribed constraints (Block D in Figure 1). If the value does not meet the constraints, it is given a very poor fitness value, after which the fitness of the configuration is compared to previous highest fitness configuration (Block E in Figure 1). The fitness of a configuration refers to its average abrasive energy value, with a higher abrasive energy being of higher fitness. If the fitness of the current configuration is higher than the previous highest fitness version, then the previous fitness version is replaced with the current input and output values, and used as the new highest fitness baseline. If the file does not have a higher fitness value, the file is discarded; however, its input and fitness values are retained by the GA. This process iterates until the population size is exceeded (Block H), after which the fittest individuals are used (with crossover and mutation operations) to seed the next generation of input files. This process of continual improvement (based on natural selection) continues until the genetic algorithm converges to an optimal solution, or until the maximum generation limit (60) is reached (Block I).

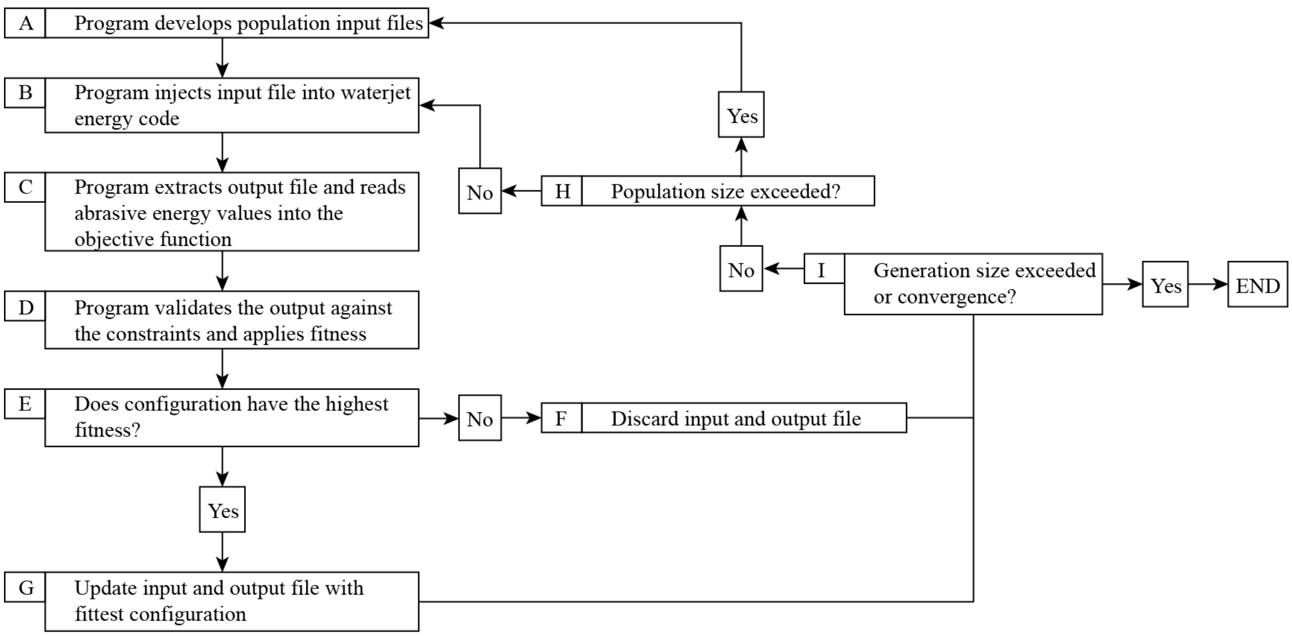

Figure 1 Program structure for genetic algorithm optimization of AWJ nozzles

2.2. Modelling abrasive particle energy

The average abrasive energy used in the genetic algorithm fitness function is approximated using the waterjet energy model developed by Narayanan et al. [30] who has shown to have an accuracy of ~10-15%. The inputs used to approximate the abrasive energy are: orifice and mixing tube diameter, mixing tube length, mixing chamber length and diameter, feed port length and diameter, water density and viscosity, air density and viscosity, ambient pressure and temperature, discharge coefficient and abrasive diameter. The output used in the objective function was the average abrasive energy. It was hypothesized that optimizing abrasive energy, rather than abrasive velocity, would result in increased separation speed, due to the trade-off between abrasive velocity and abrasive loading, with larger impact energies resulting in greater material removal.

2.3. Genetic algorithm program

GA mimic the process of natural selection - the strongest individuals in a population will have the highest fitness and therefore reproduce more children. After the initial population has been fully analyzed, the GA then selects a few of the fittest configurations and 'mates' them, taking parameters from both inputs. Additional modifications are made to selected children in the new population called mutations and crossovers, in order to ensure that the optimization solution is a global maximum and not a local one. The process of continual improvement through each generation is the foundation of GA, with the final population having very similar optimized attributes. This process of randomized continual improvement makes GA's a robust optimization routine, capable of handling complex systems with numerous variables.

The optimization program began by producing a randomized base population (input files). The input parameters were varied within the upper and lower bounds of the GA. The upper and lower bounds are user defined ranges through which the GA can select the input value. Utilizing upper and lower bounds greatly decreases the computation time and increases the model fidelity. The optimization program then injects the generated input file into the waterjet energy model, and extracts the average abrasive energy. The GA then assess the fitness of the input parameters against the objective function – eqn. (1)

$$f_O = \frac{1}{E_a} \tag{1}$$

where f_O is the fitness value and $E_a$is the average abrasive energy extracted from the waterjet energy model which is defined as,

$$E_a = \frac{1}{2}\dot{m}_a V_{Pavg} \tag{2}$$

Where $\dot{m}_a$ is the mass abrasive flow rate and V_{Pavg} is the average abrasive velocity as resolved by the waterjet energy model [30]. By design, GA's are minimization algorithms, thus the inverse results in a maximum value of E_a. As mention in Section 2.2, it is expected that increasing the abrasive energy will results in an increase in erosion. Thus, when validating the performance of the nozzle, it is expected that that optimized design can cut faster than a baseline comparison nozzle. The results of the objective function is constrained by mixing tube-to-orifice diameter ratio, mixing tube length to mixing tube diameter ratio, abrasive diameter to mixing tube diameter and hydraulic power requirements. The constraints are defined as

$$N_d \geq 2 \cdot O_d \tag{3}$$

$$N_L \geq 100 \cdot N_d \tag{4}$$

$$N_d \geq 3 \cdot p_d \tag{5}$$

$$P_{BL} \geq P \tag{6}$$

Where N_d is the mixing tube diameter, O_d is the orifice diameter, N_L is the mixing tube length, p_d is the abrasive diameter, P_{BL} is the hydraulic power baseline and P is the hydraulic power of the individual configurations. Constraints in eqn. (3) and (4) were based on the work of Liu and Schubert [31] who state that the optimal mixing tube-to-orifice ratio and optimal mixing tube diameter to mixing tube length aspect ratio is 2-3 and 100, respectively. These ratios ensure adequate acceleration and entrainment. Eqn (5) is based on the work of Mazurkiewicz [7] whom states that the optimal abrasive diameter is 2-3 times smaller than the nozzle diameter. Lastly, eqn (6) compares the hydraulic power requirement of the nozzle configuration and was approximated using eqn. (7) and (8) [31]

$$P = p \cdot \frac{Q}{c} \tag{7}$$

$$Q = c_d\, A\sqrt{\frac{2p}{\rho}} \tag{8}$$

Where p is the pump pressure, c is a constant equal to 60 when P is in kW, Q is the volumnetric flow rate, A is the orifice crossestional area, c_d is the discharge coefficient (set to 0.65 [31]) and ρ is the density of water. During the validation process, the optimized nozzle configuration is compared to a previous nozzle design. Thus, the hydraulic power was considered in order ensure that the optimized nozzle configuration operated at the same or better efficiency. These constraints can be modified/relaxed depending on the objective of the optimization.

3. Validation and Discussion

The GA program was validated based on the previous work of Liu et al. [32], who determined the abrasive velocity of a AWJ nozzle using a dual disk anemometer with the nozzle geometry and operating conditions shown in Table 1. The experimental nozzle and optimized nozzle were compared on their approximate cutting rates and power consumption. It was hypothesized that optimizing the abrasive energy, rather than abrasive velocity, would result in increased separation speed, due to the trade-off between abrasive velocity and abrasive loading.

Table 1 Experimental Nozzle and operating conditions

Parameter	Value
Pressure (MPa)	345
Abrasive flow rate (kg/min)	0.00987, 0.0198, 0.0395, 0.0750, 0.0988
Orifice Diameter (mm)	0.18
Nozzle Diameter (mm)	0.51
Mixing Tube Length (mm)	64
Mixing Chamber Length (mm)	8.255
Mixing Chamber Diameter (mm)	5.08

The experimental particle velocity results of the Liu et al. [32] are well approximated by waterjet energy model [30] (Figure 2), using the nozzle configuration and operating conditions shown in Table 1. The waterjet energy model had an average error of 0.5%, adding to the validity of the optimization program.

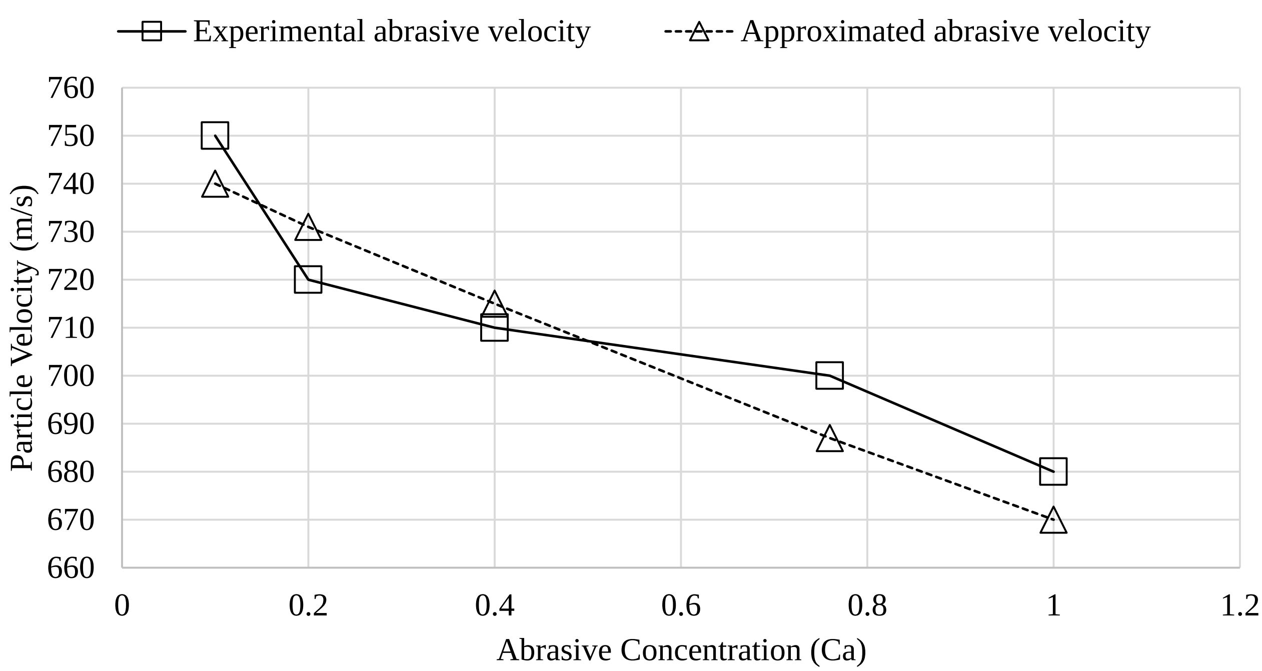

Figure 2 Comparison between the experimental and predicted particle velocity

The GA program used the nozzle geometry and configurations in Table 1 as the baseline value for the optimization. The upper and lower bounds of the GA were the product of two and ½ the baseline values, respectively. The baseline values are shown in Table 2.

Table 2 Genetic Algorithm Baseline inputs

Parameter	Baseline values
Orifice diameter (µm)	180
Mixing tube length (mm)	64
Mixing tube diameter (µm)	510
Abrasive feed pipe length (m)	0.5
Abrasive feed port diameter (mm)	1.52
Water density (kg/m^3)	1000

Water viscosity (kg/m s)	0.001
Air density (kg/m^3)	1.17
Air viscosity (kg/m s)	1.8e^{-5}
Pressure (MPa)	345
Abrasive flow rate (kg/min)	variable
Abrasive density (kg/m^3)	4000
Mixing chamber diameter (mm)	5.08
Mixing chamber length (mm)	8.25
Ambient pressure (Pa)	102700
Ambient temperature (K)	300
Discharge coefficient	0.65
Mean abrasive diameter (μm)	90

Of the input variables in Table 2, only the nozzle geometry (orifice diameter, mixing tube length, mixing tube diameter, mixing chamber diameter, mixing chamber length, abrasive feed tube length, and abrasive feed port diameter) was varied, whilst the operating conditions remained constant and the same. The objective of the optimization was program was to increase the abrasive energy exiting the mixing tube (Section 2.3), in order to produce an increase in erosion as a machining response, which translates to faster cuts. The optimization compared the results over a range of abrasive flow rates.

The optimized and baseline input configurations cutting rates were approximated using the OMAX cutting model (4th Generation) – which is based on the work of Zeng [33]. The cutting model uses operating conditions and nozzle geometry to predict the separation speed of a particular substrate based on semi-empirical relations. The geometries of both the optimized and baseline nozzle were inputted into the cutting model with the same operating conditions in order to assess which nozzle performed better. The nozzle cutting performance was compared based on their predicted separation speeds when cutting a 20mm thick 6061 Aluminum sheet for multiple abrasive concentrations. Higher separation speeds are desirable due to increase erosion rates. Figure 3 compares the separation speeds of the optimized nozzle designs and experimental nozzle, corresponding to the configurations shown in Table 3, for varying abrasive flow rates. The results show that the optimized nozzles consistently surpassed the cutting performance of the baseline nozzle with a trend of decreasing optimization capacity with increase in abrasive loading.

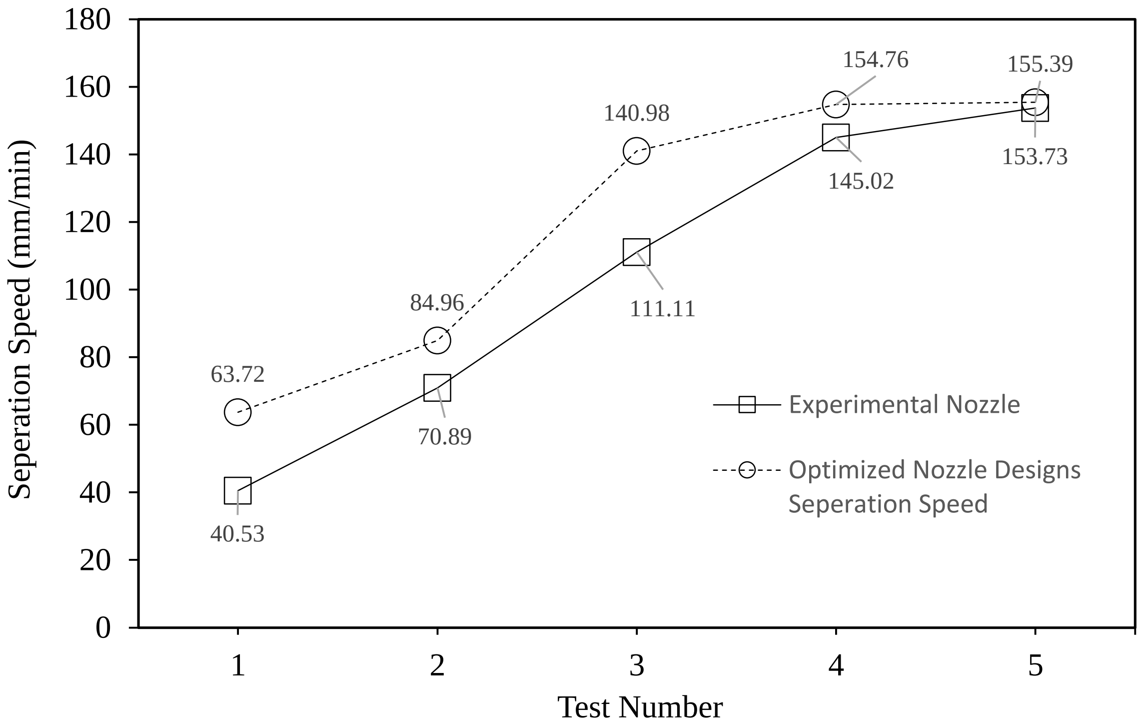

Figure 3 Comparison between separation speeds of experimental nozzle and optimized nozzle designs

Table 3 shows the optimized nozzle geometries and their respective efficiency increases. The results show that for lower abrasive flow rates, the optimization was much more substantial with a large increase in separation speed whilst requiring less hydraulic energy. The last column in Table 3 shows the average optimized nozzle design (based on all test cases). Using the average nozzle geometry, the nozzle had average increase of 15% across all abrasive loadings.

Table 3 Optimized nozzle configurations and results.

Test No.	AFR (kg/min)	Orifice diameter (μm)	Mixing tube diameter (μm)	Mixing tube length (mm)	Mixing chamber diameter (mm)	Mixing chamber length (mm)	Feed tube length (m)	Feed tube port diameter (mm)	% Diff in separation speed
1	0.00998	180.00	386.98	122.24	4.00	16.48	0.99	1.51	+36.39
2	0.01996	180.00	448.96	117.10	4.25	13.89	1.00	1.28	+16.56
3	0.03992	180.00	387.90	124.45	5.08	16.29	0.71	1.34	+21.19
4	0.07584	180.00	433.47	117.09	5.08	13.41	0.91	1.51	+6.29
5	0.09979	180.00	488.68	110.75	7.73	12.09	0.73	1.43	+1.07
Avg	-	180.00	429.20	118.33	5.23	14.43	0.87	1.41	+15.13

We can conclude that the GA nozzle optimization program is a powerful tool for nozzle design. One of the main advantages of the optimization program is its ability to resolve the numerous interacting effects of nozzle geometry and operating conditions, in order to give a pure optimized solution. Additionally, the optimization program can add/modify additional constraints in order to satisfy engineering requirements. The program can also be used to optimize operating conditions for specific nozzle designs by allowing the operating conditions to be varied rather than nozzle geometry.

4. Conclusion

This paper presents a novel method for optimizing nozzle design analytically. The foundation of the optimization routine is a waterjet energy model, capable of resolving the abrasive energies for various nozzle geometries and operating conditions. The process uses genetic algorithm optimization to vary the input parameters, in order to maximize the abrasive energies. The results of the optimization program are then compared to an experimental investigated nozzle based on cutting speed and hydraulic efficiency. The results show that the optimized nozzle designs could cut, on average, 16% faster over a range of abrasive flow rates.

5. Future Work

The next phase of the study is the prototype development of an optimized nozzle design and perform experimental testing. Additionally, the current model does not consider the effects of substrate properties and impact angle, which has been shown to influence the cutting efficiency. Implementing these effects in the optimization routine would be beneficial.

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